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DESIGNING STABLE ROCKETS

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Model rockets can be designed to be stable by using the principle of moments and centroids. The advantage of this method to model rocket builders is that it is very accurate, yet doesn't require a knowledge of higher mathematics. This accuracy has been proven in wind tunnel tests.

METHOD OF DESIGN

The first step is to decide what kind of rocket you want to design and which component parts you want to use. After this decision has been made the design is started by weighing each item to be used. For the time being, the rocket's fins are completely ignored. Using the weight of each component part we find the center of gravity of the whole rocket neglecting the fins. The next step is to determine the cross-sectional area of the components that are exposed to the freestream (for our purposes this will be the portion of the nose cone that protrudes from the body tube, the body tube and any portion of the engine that extends out of the body). With these areas we can calculate the center of pressure.

These calculations will show that the center of pressure is significantly ahead of the center of gravity. With the center of pressure ahead of the center of gravity the rocket is likely to be highly unstable--with disastrous results. In order to move the CP behind the CG we add fins. The size of the fins determines where the final CP of the rocket will be. If we decide to locate the CP 1-1/2 diameters of the body tube behind the calculated CG we can determine the exact fin area needed.

CENTER OF GRAVITY (CG)

The center of gravity of the complete rocket is found by accurately weighing each component of the rocket, determining the center of gravity of each part and then applying the principle of moments. A table (fig. 1) should be made with the various weights and the distances of the CG's from a reference line taken at the nose cone tip.

ITEMS	WEIGHT	DISTANCE OF CG FROM THE REF. LINE	WEIGHT TIMES DISTANCE
NOSE CONE	W_1	D_1	$W_1 D_1$
PARACHUTE	W_2	D_2	$W_2 D_2$
ENGINE BLOCK	W_3	D_3	$W_3 D_3$
BODY TUBE	W_4	D_4	$W_4 D_4$
LOADED ENGINE	W_5	D_5	$W_5 D_5$
TOTAL, (Σ)	ΣW		ΣWD

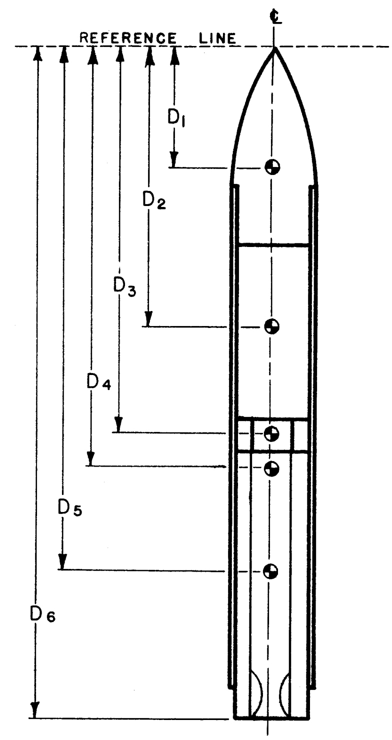
W - WEIGHT OF ITEM
D - DISTANCE OF ITEM CG FROM REFERENCE LINE
Σ - GREEK LETTER SIGMA, MEANING THE TOTAL

CENTER OF GRAVITY = (CG)

$$CG = \frac{\Sigma WD}{\Sigma W}$$

Fig. 1

All of the weights in the column labeled WEIGHTS should be added up. This total is referred to as the "Weight Summation" or simply the total weight. The Greek letter Sigma (Σ) is commonly used to indicate summation, or the total.



D_1 - DISTANCE TO CG OF NOSE CONE
D_2 - DISTANCE TO CG OF PARACHUTE
D_3 - DISTANCE TO CG OF ENGINE BLOCK
D_4 - DISTANCE TO CG OF BODY TUBE
D_5 - DISTANCE TO CG OF LOADED ENGINE
D_6 - TOTAL LENGTH OF ROCKET

CENTER OF GRAVITY

NEGLECTING THE FINS

Fig. 2

The distance from the reference line to each individual CG is the next item to determine. This can be found by balancing the item on a knife edge and then measuring this balance point's distance to the furthest forward point of the component. This distance is added to the distance between the reference line and the forward point on the part to find the item's CG distance from the reference line.

Next, multiply the weight by the distance and put the result in the column labeled "WEIGHT TIMES DISTANCE." This must be done for each item. All of the different products are then added up. This total is called the "Moment Summation, (ΣWD).

Finally the values obtained from the table are substituted into the formula shown in fig. 1. Divide the WD summation by the W summation to find the CG distance from the reference line.

CENTER OF PRESSURE (CP)

The center of pressure is determined by accurately calculating the cross-sectional area of the components of the rocket which are exposed to the freestream in flight. Fig. 3 illustrates the portions of the rocket to be considered. Part of the nose cone is inserted into the body tube and is not used in this calculation. All of the body tube is exposed so we use all of it. The engine usually sticks out 1/4 to 1/2 inch and the part that sticks out must be included. You will notice that we are still not considering the fins.

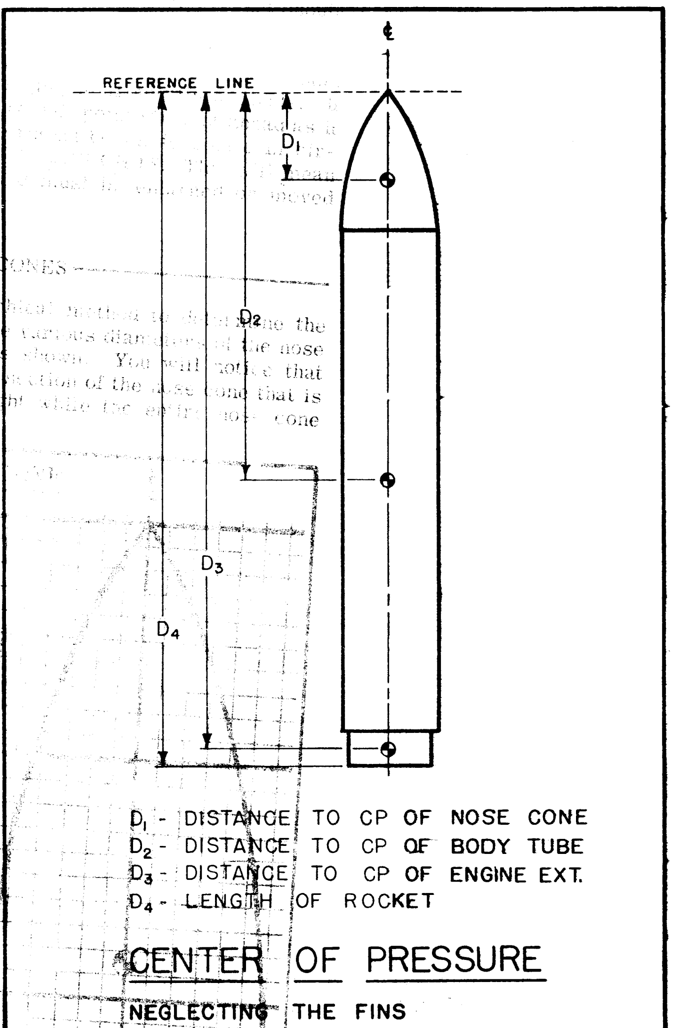


Fig. 3

All of these areas should be listed in the column labeled "AREA." Add up all of these areas to get the "Area Summation, (ΣA)."

The distance from the reference line to each individual CP is the next item to determine. There is a section at the end of this report that will show you how to determine this for the nose cone. The CP of a rectangle (body tube cross-section) always lies exactly half way between its sides (1/2 of the length of the body tube). This distance is added to the exposed length of the nose cone to give you the distance from the reference line to the body CP. The exposed section of the engine is also a rectangle. Its CP is exactly 1/2 of the exposed length. The exposed engine CP is then added to the body tube length and the exposed nose cone length to give its CP distance from the reference line.

Finally the values obtained from the table are substituted into the formula shown in fig. 4. By dividing the AD summation by the A summation the CP distance from the reference line is calculated.

ITEMS	AREA	DISTANCE OF CP FROM THE REF. LINE	AREA TIMES DISTANCE
NOSE CONE	A_1	D_1	$A_1 D_1$
BODY TUBE	A_2	D_2	$A_2 D_2$
ENGINE EXTENSION	A_3	D_3	$A_3 D_3$
TOTAL, (Σ)	Σ A		Σ AD

A - AREA OF ITEM

D - DISTANCE OF ITEM CP FROM REFERENCE LINE

Σ - GREEK LETTER SIGMA, MEANING THE TOTAL

CENTER OF PRESSURE = (CP)

$$CP = \frac{\Sigma AD}{\Sigma A}$$

Fig. 4

FIN AREA DETERMINATION

Figure 5 illustrates the points of the CP and CG that we calculated. Also shown is the position of the stable CP and the arbitrary point that we have chosen for the center of pressure of the fins. This arbitrary fin CP can be chosen at any point behind the stable CP. The illustration shows a symmetrical rectangular fin which is the easiest to use. Swept back fins will have their CP at a point farther behind the stable CP. When the fin CP is moved farther behind the stable CP the required

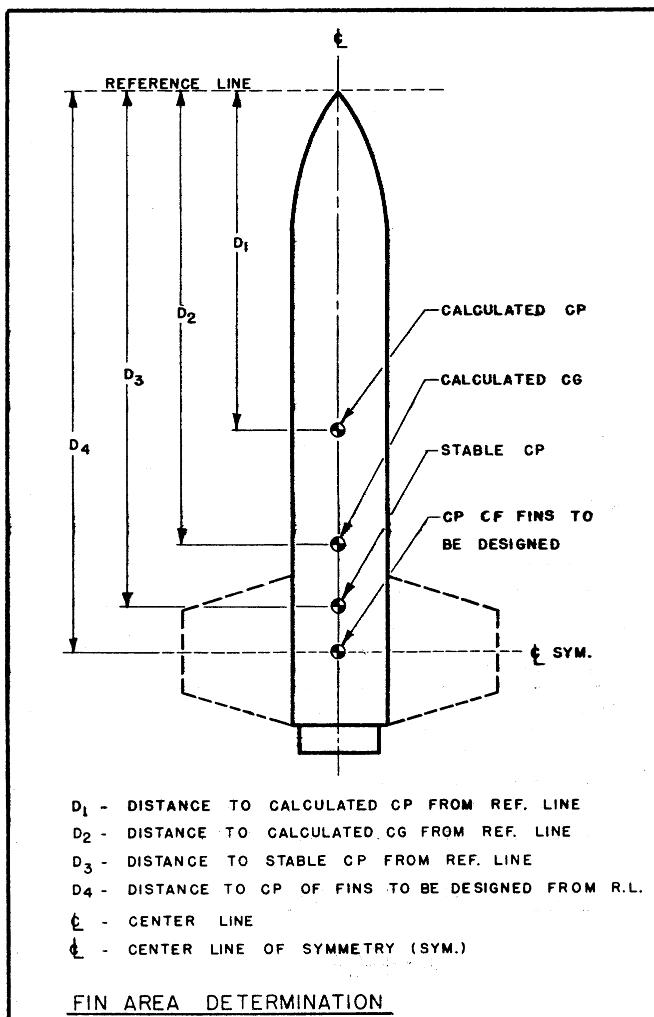


Fig. 5

fin area becomes smaller. Consequently drag decreases and the rocket becomes lighter which increases its potential altitude.

The formula presented in fig. 6 is used to determine the necessary fin cross-sectional area. The example is given to show the method of finding the fin area when all other quantities are known. When the design has reached this step the values for ΣA and ΣAD will have been calculated and the CP and Dcp will have been decided. The only unknown quantity left is fin cross-sectional area which is found by the method shown.

$$CP = \frac{\Sigma AD + (D_{CP} \times A_{FIN})}{\Sigma A + A_{FIN}}$$

ΣAD - SEE FIG. 4 FOR CALCULATED VALUE

ΣA - SEE FIG. 4 FOR CALCULATED VALUE

CP - DISTANCE FROM REF. LINE TO THE STABLE CP (FIG. 5), 1-1/2 DIAMETERS BEHIND THE CALCULATED CG (FIG. 2)

D_{CP} - DISTANCE FROM REF. LINE TO THE DESIGN POINT FOR FIN CP

A_{FIN} - AREA OF FINS

EXAMPLE:

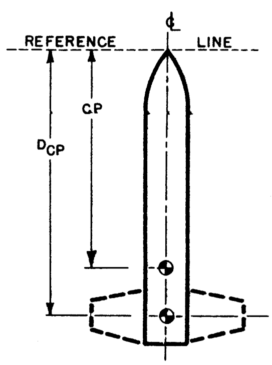
$\Sigma A = 16\text{-IN}^2$

$\Sigma AD = 64\text{-IN}^3$

$D_{CP} = 7\text{-IN}$

$CP = 6\text{-IN}$

A_{FINS} = UNKNOWN



$$6 = \frac{64 + (7 \times A_{FIN})}{16 + A_{FIN}}$$

$$6(16 + A_{FIN}) = 64 + 7A_{FIN}$$

$$96 + 6A_{FIN} = 64 + 7A_{FIN}$$

$$96 - 64 = 7A_{FIN} - 6A_{FIN}$$

$$\boxed{32 = A_{FIN}}$$

FIN AREA DETERMINATION Fig. 6

The stable CP is located 1-1/2 body tube diameters behind the calculated CG because the fins with their slight weight will move the calculated CG to the rear of the rocket a small amount. The distance of 1-1/2 body tube diameters is sufficient to allow for this CG movement and maintain the stable CP at least 1 body tube diameter behind the final loaded rocket CG. If unusually large fins are used then a larger distance between calculated CG and stable CP must be used to allow for the greater weight of the fins. Most model rockets can be successfully designed by using the 1-1/2 body tube diameter distance.

FIN AREA DETERMINATION CONTINUED

The example in fig. 5 shows how to calculate the cross-sectional area required. If a four-finned rocket is designed then this calculated area must be divided by 2 for the area of one fin. If a three-finned rocket is desired, this calculated area must be divided by 1.5 for the area of one fin.

The final CG of the rocket can now be calculated. The method for doing this is exactly like the method shown in fig. 2. This time we add the total weight of all the fins and add this value in the column labeled WEIGHT. The total fin weight is then multiplied by the distance from the reference line to the CG of the fins and this product is put in the column labeled WEIGHT TIMES DISTANCE. Then, as before, we add up the weight of all items, this time including the weight of the fins, to get the WD summation. The WD summation is now divided by the W summation to yield the final CG distance from the reference line.

To be as accurate as possible the weight of the finish and the weight of the glue used can be included; however, this is not necessary unless you want extreme accuracy. If these weights are included, it is best to use the dry weights of the materials rather than the weights of the liquid material.

The final CG and CP are now calculated and the rocket should be stable in flight. To be certain that you have not made a mistake with your calculations you should check the rocket's stability after it has been built by tying a string around the model at its center of gravity and swinging it around in a circle. If the rocket is stable the nose cone will point straight ahead as it moves around. If, however, the rocket begins to rotate in circles around the string then DON'T LAUNCH IT. This will mean that your fins are too small and must be enlarged or moved farther back.

NOSE CONES

Figure 7 illustrates the graphical method to determine the CP of a nose cone. Measure the various diameters of the nose cone and plot them on a graph as shown. You will notice that the CP graph includes only the section of the nose cone that is exposed to the freestream in flight while the entire nose cone length is used in finding its CG.

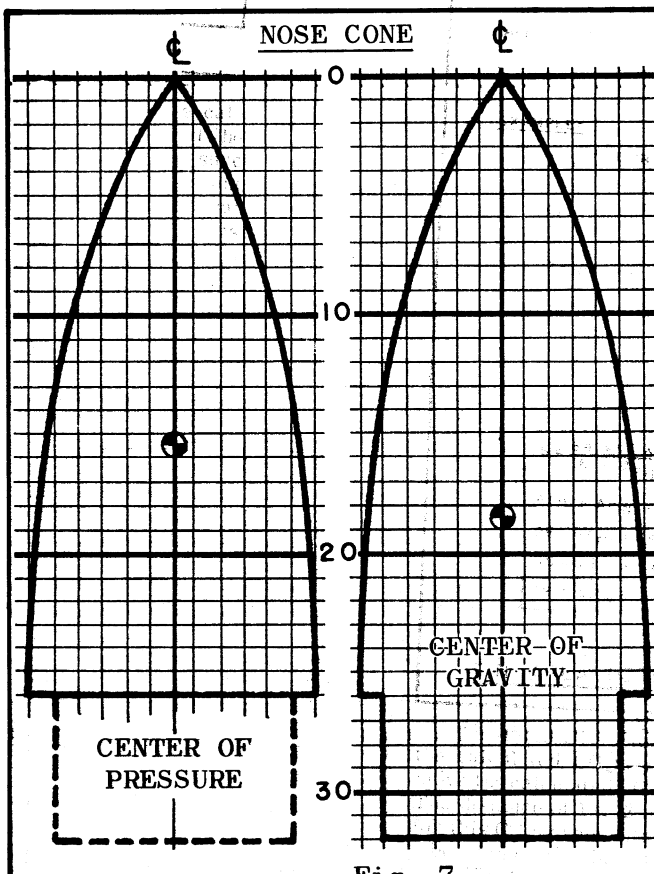


Fig. 7

Each square shown is 1/10 inch long and 1/10 inch wide giving an area of 1/100 square inch. The cross-sectional area is found by counting the total number of squares enclosed. The total number of squares should be divided by 100 to give the area in square inches. The center of pressure will be at a point where exactly one half of the area is on each side of the line drawn perpendicular to the center line of the nose cone.

VARIOUS TYPES OF FINS

Figures 8 and 9 illustrate two of the most widely used types of fins. The illustrations and the following method for the solution of CP and CG may be used with any dimensions. The actual method of solution is illustrated below in fig. 8A and 9A. It looks familiar, doesn't it? This principle of centroids and moments is very important in the design of rockets.

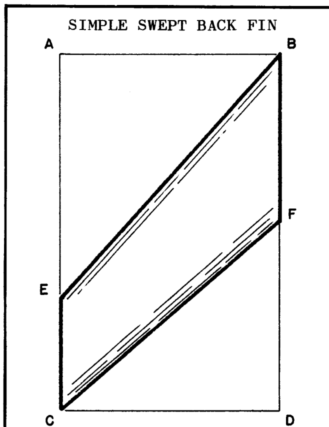


Fig. 8

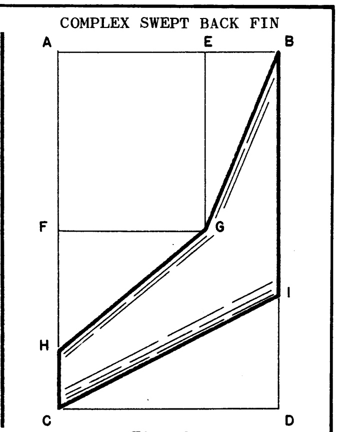


Fig. 9

Figures 8A and 9A illustrate the tabular solution for the CP of the tail fins. The idea here is to find the CP of the square (ABCD) and then to subtract the CP of the smaller right triangles and the smaller square. This leaves the CP of the fin. The CP and CG of the fin are considered to be located at exactly the same spot so that when you calculate the fin CP you also have the fin CG. Notice the plus and minus signs in fig. 8A and 9A. The plus sign is used for the area of the large square that encloses the fin. The minus sign used in the smaller triangles and the smaller square simply means that these CP's are not part of the fin. To get the summation A, (ΣA), add all of the areas that have a minus sign and subtract this total from the area with the plus sign.

SIMPLE SWEPT BACK FINS

ITEM	AREA	CP DISTANCE FROM THE REFERENCE LINE	AREA TIMES DISTANCE
ABCD	AB X BD (+)	1/2 BD	(+)
ABE	1/2(AB)(AE) (−)	1/3 AE	(−)
CDF	1/2(CD)(DF) (−)	BF + 2/3(DF)	(−)
	Σ A		Σ AD

$$CP = \frac{\Sigma AD}{\Sigma A}$$

Fig. 8A

COMPLEX SWEPT BACK FINS

ITEM	AREA	CP DISTANCE FROM THE REFERENCE LINE	AREA TIMES DISTANCE
ABCD	AB X BD (+)	1/2 BD	(+)
AEGF	AE X EG (−)	1/2 AF	(−)
BEG	1/2(BE X EG) (−)	1/3 EG	(−)
FGH	1/2(FG X FH) (−)	AF + 1/3 FH	(−)
CDI	$(DI \times CD)\frac{1}{2}$ (−)	BI + 2/3 ID	(−)
	Σ A		Σ AD

$$CP = \frac{\Sigma AD}{\Sigma A}$$

Fig. 9A

The column labeled AREA TIMES DISTANCE also has plus and minus signs. The moment will have a minus sign when the area, with a minus sign, is multiplied by a distance. To get the summation AD, (ΣAD), add all of the terms that have a minus sign and subtract this total from the area with the plus sign. The summation A and the summation AD are substituted into the illustrated formula.

Figure 10 represents a tail design that can not practically be designed by the moment and centroid method. It requires the same type of procedure used on the nose cone. Count the total number of squares enclosed by the fin. The illustrated graph has ten lines to the inch which is 100 squares in 1 square inch. To get the area you simply divide the total number of squares by the number of squares in a square inch. (When you do this be certain how many lines to the inch your graph paper has. Many have 8 lines to the inch and some have 20.)

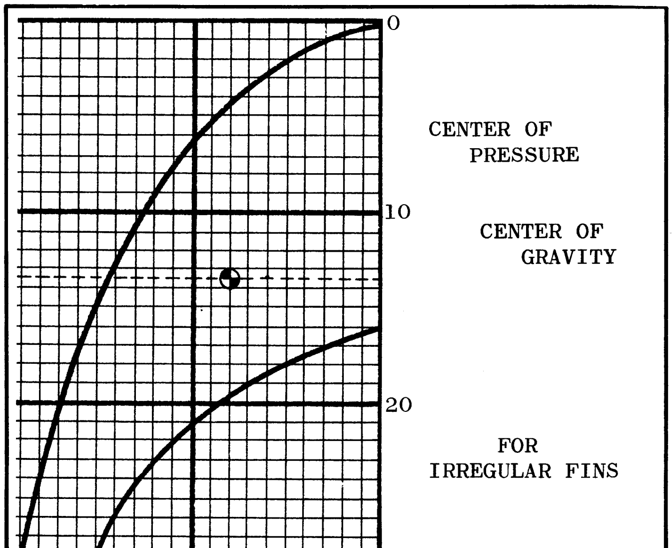


Fig. 10

The next step is to begin at the top of the fin and count the squares horizontally until you find out where exactly half of the area is. You will probably end up half way across a row of squares. To be very accurate make a note of the total number of squares in that row and also the number that you counted. For example, if there are a total of 15 squares in that horizontal row and you counted 5 squares before you came to half of the total area, then the CP will lie a distance of 5/15 or 1/3 between the two lines.

REFERENCES:

For a more detailed explanation of the principle of moments and centroids refer to the following books. There are illustrated problems worked in each of them.

ENGINEERING MECHANICS, 2nd Edition, by Ferdinand L. Singer, Chapter VII, Harper and Brothers, New York; 1954

THEORY AND PROBLEMS OF ENGINEERING MECHANICS, by W. G. McLean and E. W. Nelson, Chapter 9, Schaum Publishing Company; 1952